Lesson 5 Classical Monetary Model (2)

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6 Equilibrium under Log-linearization

- Combining Eq.(3.24)-(3.24) yields:

$$n_t = 0 \quad (3.25)$$
$$y_t = a_t \quad (3.26)$$

- Eqs.(3.25) and (3.26) show that employment and income are determined egeogenously.
- Employment is constant while income increases if productivity increases, and vice varca.

- Eq.(3.21) can be rewritten as:

$$\hat{r}_t = \mathrm{E}_t(\Delta y_{t+1})$$

where $\hat{r}_t \equiv \hat{i}_t - \mathrm{E}_t(\pi_{t+1})$ denotes the percentage deviation of real interest rate from its steady state value, Δ denotes the operator of logarithmic differential. Then, $\Delta v_t \equiv v_t - v_{t-1}$ denotes the percentage changes in any variables V_t.
- Plugging Eq.(3.26) into this yields:

$$\hat{r}_t = \frac{1}{\alpha}\mathrm{E}_t(\Delta a_{t+1}) \quad (3.27)$$

- Plugging Eq.(3.25) into Eq.(3.23), we get:

$$w_t - p_t = a_t \quad (3.28)$$

which shows that the real wage is an increasing function of the productivity.
- Eqs.(3.26) and (3.28) implies that an increase in the productivity increases the income through an increase in the real wage, and vice versa.
- In this economy, an increase in the real wage increases real wage immediately, and vice versa

7 Monetary Policy Neutrality

- Suppose that the nominal interest rate i_t is monetary policy instrument.
- Eqs.(3.25)--(3.26) show that the GDP, the employment and the real interest rate are determined by the productivity.
- On the other hand, those equalities show that these are not determined by the nominal interest rate.
- Thus, real variables are independent from monetary policy.
- That is, monetary policy is neutral in this economy.

8 Determination on Nominal Variables

- Alright the, how nominal variables such as the inflation and the nominal interest rate are determined?
- These are determined apart from real variables.
- Now, we consider how these are determined under various monetary policy rules.
- Under any rules, we assume Fischer equation is apllied as follows:

$$\hat{i}_t = \hat{r}_t + \mathrm{E}_t(\pi_{t+1}) \quad (3.30)$$

8.1 The Case where the Nominal interest Rate is Determined Exogenously

- Suppose that the nominal interest rate is determined exogenously as follows:

$$\hat{i}_t = 0 \qquad (3.31)$$

- This rule implies $i_t = i = \delta$ immediately. Thut is, the nominal interest rate equals to the rate of time preference and is constant overtime.

- Plugging Eqs.(3.27) and (3.31) into Eq.(3.30) yields:

$$\mathrm{E}_t(\pi_{t+1}) = -\frac{1}{\alpha}\mathrm{E}_t(\Delta a_{t+1}) \qquad (3.31)$$

which shows that the expected inflation is determined by the productivity a_t. That is, similar to real variables, the expected inflation determined uniquely.
- How about actual inflation? Actual inflation does not necessarily equal to the expected inflation and it may accompany with prediction error. This implies that

$$\pi_{t+1} = \mathrm{E}_t(\pi_{t+1}) + \xi_{t+1}$$

where ξ_t denotes the prediction error, which is often dubbed sunspot shock. Note that $\mathrm{E}_t(\xi_{t+1}) = 0$.

- The, actual inflation is given by:

$$\pi_{t+1} = -\frac{1}{\alpha}\mathrm{E}_t(\Delta a_{t+1}) + \xi_{t+1}$$

which shows that actual inflation is indeterminate as long as it accompanies with the prediction error.
- Further, because of the definition of inflation, this equality can be rewritten as:

$$p_{t+1} = p_t - \frac{1}{\alpha}\mathrm{E}_t(\Delta a_{t+1}) + \xi_{t+1}$$

which shows that the price (level) also indeterminate as long as it accompanies with the prediction error.

- These facts show that the inflation and the price are indeterminate under the monetary policy where the nominal interest rate is determined exogenously.
- Because of prediction error, both the inflation and the price are determined uniquely.
- If the role of monetary policy is stabilizing prices, the monetary policy where the nominal interest rate is determined exogenously is not desirable.
- This fact implies that the money supply is also indeterminate.

- Suppose that the demand function for the real money balance is given by:

$$M_t / P_t = Y_t i_t^{-\eta}$$

where M_t denotes the money supply and η denotes the semi-elasticity of the demand for the real money balance.
- By log-linearizing this demand function, we have:

$$m_t - p_t = y_t - \eta \hat{i}_t$$

While the GDP is determined uniquely and the nominal interest rate is determined uniquely because of constant, the price is indeterminate.

- Thus, if the demand function for the real money balance is applied, the money supply is indeterminate.
- Similarly, Eq.(3.28) shows that the nominal wage is indeterminate if the price is indeterminate although the real wage is determinate.

8.2 Taylor Rule

- Generally, Taylor Rule is monetary policy rule that the nominal interest rate reacts to the GDP gap and the inflation.
- Here, suppose that Taylor rule is a rule that the nominal interest rate just reacts to the inflation as follows:

$$\hat{i}_t = \phi_\pi \pi_t \quad (3.32)$$

where $\phi_\pi \geq 0$ denotes an reaction coefficient of the nominal interest rate to the inflation.
- Plugging Eq.(3.32) into Eq.(3.30) yields:

$$\phi_\pi \pi_t = \hat{r}_t + \mathrm{E}_t(\pi_{t+1}) \quad (3.33)$$

- We consider both cases of $\phi_\pi > 1$ and $\phi_\pi < 1$ in the following.

8.2.1 The Case of φ_π<1

- Under the case of $\phi_\pi < 1$, The solution of Eq.(3.33) is given by:

$$\pi_{t+1} = \phi_\pi \pi_t + \xi_{t+1} - \hat{r}_t \quad (3.34)$$

- Sunspot shock appears in Eq.(3.34) and the inflation is indeterminate.

8.2.1 The Case of φ_π＞1

- Next, we consider the case of $\phi_\pi > 1$.
- Eq.(3.33) can be rewritten as:

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} \mathrm{E}_t(\hat{r}_{t+k}) \quad (3.35)$$

- Which shows that current inflation equals to the sum of the net present value of the real interest rate.
- As shown in Eq.(3.27), the real interest rate is determined uniquely. Thus, the inflation is also determined uniquely.
- Because of $1/\phi_\pi < 1$, the inflation stays extremely around its steady state and is close to zero. 。

Proof of Eq.(3.34)

- Rearranging Eq.(3.33) as follows:

$$\begin{aligned}\pi_t &= \phi_\pi^{-1}\hat{r}_t + \phi_\pi^{-2}\mathrm{E}_t(\hat{r}_{t+1}) + \phi_\pi^{-3}\mathrm{E}_t(\hat{r}_{t+2}) + \cdots \\ &= \phi_\pi^{-1}\hat{r}_t + \sum_{k=1}^{\infty}\phi_\pi^{-(k+1)}\mathrm{E}_t(\hat{r}_{t+k})\end{aligned} \quad (3.36)$$

- By leading Eq.(3.36) one period forward, we get:

$$\mathrm{E}_t(\pi_{t+1}) = \phi_\pi^{-1}\mathrm{E}_t(\hat{r}_{t+1}) + \phi_\pi^{-2}\mathrm{E}_t(\hat{r}_{t+2}) + \phi_\pi^{-3}\mathrm{E}_t(\hat{r}_{t+3}) + \cdots$$

- Multiplying bith sides of this by ϕ_π^{-1} yields:

$$\begin{aligned}\phi_\pi^{-1}\mathrm{E}_t(\pi_{t+1}) &= \phi_\pi^{-2}\hat{r}_{t+1} + \phi_\pi^{-3}\mathrm{E}_t(\hat{r}_{t+2}) + \phi_\pi^{-4}\mathrm{E}_t(\hat{r}_{t+3}) + \cdots \\ &= \sum_{k=1}^{\infty}\phi_\pi^{-(k+1)}\mathrm{E}_t(\hat{r}_{t+k})\end{aligned} \quad (3.37)$$

- Plugging Eq.(3.37) into Eq.(3.36) yields:

$$\pi_t = \phi_\pi^{-1}\hat{r}_t + \phi_\pi^{-1}\mathrm{E}_t(\pi_{t+1})$$

- Multiplying bothsides of this by ϕ_π yields:

$$\phi_\pi \pi_t = \hat{r}_t + \mathrm{E}_t(\pi_{t+1})$$

which is Eq.(3.33) itself.

Q.E.D.

- Now, we assume a_t follows AR(1) process as follows:

$$a_t = \rho a_{t-1} + \varepsilon_t^a \quad (3.38)$$

where $\rho \in [0,1)$ denotes an auto regressive coefficient of the productivity and ε_t^a is an error term which is i.i.d. and suffices $\mathrm{E}_t(\varepsilon_{t+1}^a) = 0$.
- Plugging this and Eq.(3.27) into Eq.(3.35) yields:

$$\pi_t = -\frac{1-\rho}{\alpha(\phi_\pi - \rho)} a_t \quad (3.39)$$

which shows that the inflation is determined uniquely similar to real variables.

Proof of Eq.(3.38)

- Plugging Eq.(3.27) into Eq.(3.38) yields:

$$\hat{r}_t = -\frac{1-\rho}{\alpha} a_t \tag{3.40}$$

- By leading Eq.(3.40) one period forward yields:

$$\mathrm{E}_t\left(\hat{r}_{t+1}\right) = -\frac{(1-\rho)\rho}{\alpha} a_t \tag{3.41}$$

- By leading Eq.(3.40) two period forward yields:

$$\mathrm{E}_t\left(\hat{r}_{t+2}\right) = -\frac{(1-\rho)\rho^2}{\alpha} a_t \tag{3.42}$$

- Plugging Eq.(3.40)--(3.42) into Eq.(3.35) yields:

$$\begin{aligned}\pi_t &= -\phi_\pi^{-1}\frac{1-\rho}{\alpha}a_t - \phi_\pi^{-2}\frac{(1-\rho)\rho}{\alpha}a_t - \phi_\pi^{-3}\frac{(1-\rho)\rho^2}{\alpha}a_t - \cdots \\ &= a_t\left(-\frac{1-\rho}{\alpha}\right)\left(\phi_\pi^{-1} + \rho\phi_\pi^{-2} + \rho^2\phi_\pi^{-3} + \cdots\right) \\ &= a_t\left(-\frac{1-\rho}{\alpha}\right)\frac{1}{\rho}\left(\rho\phi_\pi^{-1} + \rho^2\phi_\pi^{-2} + \rho^3\phi_\pi^{-3} + \cdots\right) \\ &= a_t\left(-\frac{1-\rho}{\alpha\rho}\right)\left[\frac{\rho}{\phi_\pi} + \left(\frac{\rho}{\phi_\pi}\right)^2 + \left(\frac{\rho}{\phi_\pi}\right)^3 + \cdots\right]\end{aligned}$$

- Let define $\tilde{a}_t \equiv a_t\left[-(1-\rho)/(\alpha\rho)\right]$ and deviding both sides of this by $\tilde{a}_t$ yields:

$$\frac{\pi_t}{\tilde{a}_t} = \frac{\rho}{\phi_\pi} + \left(\frac{\rho}{\phi_\pi}\right)^2 + \left(\frac{\rho}{\phi_\pi}\right)^3 + \cdots \tag{3.43}$$

- By multiplying ρ/ϕ_π both sides of Eq.(3.43), we have:

$$\frac{\pi_t}{\tilde{a}_t}\frac{\rho}{\phi_\pi} = \left(\frac{\rho}{\phi_\pi}\right)^2 + \left(\frac{\rho}{\phi_\pi}\right)^3 + \left(\frac{\rho}{\phi_\pi}\right)^4 \cdots \tag{3.44}$$

- Subtracting Eq.(3.44) from Eq.(3.44) yields:

$$\frac{\pi_t}{\tilde{a}_t}\left(1-\frac{\rho}{\phi_\pi}\right) = \frac{\rho}{\phi_\pi}$$

- By deviding $\left(1-\frac{\rho}{\phi_\pi}\right)\frac{1}{\tilde{a}_t}$ both sides of this yields:

$$\pi_t = -\frac{1-\rho}{\alpha(\phi_\pi-\rho)}a_t$$

which is Eq.(3.39) itself.

Q.E.D.

- When $\phi_\pi > 1$, the inflation is determined uniquely.
- This condition is dubbed Taylor Principle.
- If Taylor principle is applied, the nominal interest rate is hiked beyond one unit to one unit increase in the inflation.

8.3 The Case where the Money Supply is Determined Exogenously

- Now, we consider the case where the money supply is determined exogenously.
- Plugging Eq.(3.30) into the demand function for the real money balance $m_t - p_t = y_t - \eta\hat{i}_t$ yields:

$$p_t = \frac{\eta}{1+\eta}\mathrm{E}_t\left(p_{t+1}\right) + \frac{1}{1+\eta}m_t + u_t \tag{3.45}$$

where $u_t \equiv (1+\eta)^{-1}\left(\eta r_t - y_t\right)$ denotes a block of variables independent from monetary policy.

- Suppose that $\eta > 0$. By solving Eq.(3.45) foeward, we get:

$$p_t = \frac{1}{1+\eta}\sum_{k=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(m_{t+k}\right)+u_t^{'} \quad (3.46)$$

where $u_t^{'} \equiv \sum_{k=0}^{\infty}\left[\eta/(1+\eta)\right]^k \mathrm{E}_t\left(u_{t+k}\right)$ denotes a term independent from monetary policy.

- Eq.(3.46) can be rewritten as:

$$p_t = m_t + \sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(\Delta m_{t+k}\right)+u_t^{'} \quad (3.47)$$

which shows that the price is determined uniquely. Note that the expectation of the prediction error is zero.

Proof of Eq.(3.46)

- Eq.(3.46) can be rewritten as:

$$p_t = \frac{1}{1+\eta}\left[m_t + \frac{1}{1+\eta}m_{t+1} + \left(\frac{1}{1+\eta}\right)^2 m_{t+2} + \cdots\right]$$

$$+u_t + \frac{1}{1+\eta}u_{t+1} + \left(\frac{1}{1+\eta}\right)^2 u_{t+2} + \cdots$$

$$= \frac{1}{1+\eta}m_t + \frac{1}{1+\eta}\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(m_{t+k}\right) + \sum_{k=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(u_{t+k}\right) \quad (3.47)$$

- By forwarding Eq.(3.47) one period, we have:

$$\mathrm{E}_t\left(p_{t+1}\right) = \frac{1}{1+\eta}\mathrm{E}_t\left[m_{t+1} + \frac{\eta}{1+\eta}m_{t+2} + \left(\frac{\eta}{1+\eta}\right)^2 m_{t+3} + \cdots\right]$$

$$+\mathrm{E}_t\left[u_{t+1} + \frac{\eta}{1+\eta}u_{t+2} + \left(\frac{\eta}{1+\eta}\right)^2 u_{t+3} + \cdots\right]$$

$$= \frac{1}{1+\eta}\frac{1+\eta}{\eta}\mathrm{E}_t\left[\frac{\eta}{1+\eta}m_{t+1} + \left(\frac{\eta}{1+\eta}\right)^2 m_{t+2} + \left(\frac{\eta}{1+\eta}\right)^3 m_{t+3} + \cdots\right]$$

$$+\frac{1+\eta}{\eta}\mathrm{E}_t\left[\frac{\eta}{1+\eta}u_{t+1} + \left(\frac{\eta}{1+\eta}\right)^2 u_{t+2} + \left(\frac{\eta}{1+\eta}\right)^3 u_{t+3} + \cdots\right]$$

$$= \frac{1}{\eta}\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(m_{t+k}\right) + \frac{1+\eta}{\eta}\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(u_{t+k}\right) \quad (3.48)$$

- By arranging the second term on the RHS in Eq.(3.48), we have:

$$\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(m_{t+k}\right) = \eta \mathrm{E}_t\left(p_{t+1}\right) - (1+\eta)\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(u_{t+k}\right)$$

(3.49)

- Plugging Eq.(3.49) into Eq.(3.47) yields:

$$p_t = \frac{1}{1+\eta}m_t + \frac{1}{1+\eta}\left[\eta \mathrm{E}_t\left(p_{t+1}\right) - (1+\eta)\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(u_{t+k}\right)\right]$$

$$+\sum_{k=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(u_{t+k}\right)$$

- Further, this can be rewritten as:

$$p_t = \frac{1}{1+\eta}m_t + \frac{\eta}{1+\eta}\mathrm{E}_t\left(p_{t+1}\right)$$

$$-\mathrm{E}_t\left[\frac{\eta}{1+\eta}u_{t+1} + \left(\frac{\eta}{1+\eta}\right)^2 u_{t+2} + \left(\frac{\eta}{1+\eta}\right)^3 u_{t+3} + \cdots\right]$$

$$+u_t + \mathrm{E}_t\left[\frac{\eta}{1+\eta}u_{t+1} + \left(\frac{\eta}{1+\eta}\right)^2 u_{t+2} + \left(\frac{\eta}{1+\eta}\right)^3 u_{t+3} + \cdots\right]$$

$$= \frac{1}{1+\eta}m_t + \frac{\eta}{1+\eta}\mathrm{E}_t\left(p_{t+1}\right) + u_t$$

Q.E.D.

- The forth line in this equality is obviously Eq. (3.45).

Proof of Eq.(3.47)

- Arranging Eq.(3.47) yields:

$$p_t = m_t + \sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \Delta \mathrm{E}_t\left(m_{t+k}\right) + u_t'$$
$$= m_t + \frac{\eta}{1+\eta}\left[\mathrm{E}_t\left(m_{t+1}\right) - m_t\right] +$$
$$\left(\frac{\eta}{1+\eta}\right)^2 \mathrm{E}_t\left(m_{t+2} - m_{t+1}\right) +$$
$$\left(\frac{\eta}{1+\eta}\right)^3 \mathrm{E}_t\left(m_{t+3} - m_{t+2}\right) + \cdots + u_t'$$

$$= \left(1 - \frac{\eta}{1+\eta}\right) m_t + \left[\frac{\eta}{1+\eta} - \left(\frac{\eta}{1+\eta}\right)^2\right] \mathrm{E}_t\left(m_{t+1}\right)$$
$$+ \left[\left(\frac{\eta}{1+\eta}\right)^2 - \left(\frac{\eta}{1+\eta}\right)^3\right] \mathrm{E}_t\left(m_{t+2}\right)$$
$$+ \left[\left(\frac{\eta}{1+\eta}\right)^3 - \left(\frac{\eta}{1+\eta}\right)^4\right] \mathrm{E}_t\left(m_{t+3}\right) + \cdots + u_t'$$

$$= \frac{1}{1+\eta}\left\{ m_t + \left(\eta - \frac{\eta^2}{1+\eta}\right)\mathrm{E}_t\left(m_{t+1}\right) + \left[\frac{\eta^2}{1+\eta} - \frac{\eta^3}{(1+\eta)^2}\right]\mathrm{E}_t\left(m_{t+2}\right) + \left[\frac{\eta^3}{(1+\eta)^2} - \frac{\eta^4}{(1+\eta)^3}\right]\mathrm{E}_t\left(m_{t+3}\right) + \cdots \right\} + u_t'$$
$$= \frac{1}{1+\eta}\left[m_t + \frac{(1+\eta)\eta - \eta^2}{1+\eta}\mathrm{E}_t\left(m_{t+1}\right) + \left(\frac{(1+\eta)\eta^2 - \eta^3}{(1+\eta)^2}\right)^2 \mathrm{E}_t\left(m_{t+2}\right) + \left(\frac{(1+\eta)\eta^3 - \eta^4}{(1+\eta)^3}\right)^3 \mathrm{E}_t\left(m_{t+3}\right) + \cdots \right] + u_t'$$

$$= \frac{1}{1+\eta}\left[m_t + \frac{\eta}{1+\eta}\mathrm{E}_t\left(m_{t+1}\right) + \left(\frac{\eta}{1+\eta}\right)^2 \mathrm{E}_t\left(m_{t+2}\right) + \left(\frac{\eta}{1+\eta}\right)^3 \mathrm{E}_t\left(m_{t+3}\right) + \cdots \right] + u_t'$$

$$= \frac{1}{1+\eta}\left[m_t + \frac{\eta}{1+\eta}\mathrm{E}_t\left(m_{t+1}\right) + \left(\frac{\eta}{1+\eta}\right)^2 \mathrm{E}_t\left(m_{t+2}\right) + \left(\frac{\eta}{1+\eta}\right)^3 \mathrm{E}_t\left(m_{t+3}\right) + \cdots \right] + u_t'$$

- Thus, we get:

$$p_t = \frac{1}{1+\eta}\sum_{k=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(m_{t+k}\right) + u_t'$$

which is Eq.(3.46) itself.

- By plugging Eq.(3.47) into the demand function for the real money balance, we get:

$$i_t = \frac{1}{\eta}\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(\Delta m_{t+k}\right) + u_t''$$

where $u_t'' \equiv \eta^{-1}\left(u_t' + y_t\right)$. Thus, the nominal interest rate is determined exogenously.

- Now, we assume that the growth rate of money supply Δm_t follows AR(1) process as follows:

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

where ε_t^m denotes the money supply growth rate shock which suffices $\mathrm{E}_t\left(\varepsilon_{t+1}^m\right) = 0$.

- We have already understood that both the real interest rate and the GDP are determined uniquely. Thus, for simplicity loss of generality, we suppose that the productivity is constant overtime (Here, $r_t = y_t = 0$ is applied).

- When the growth rate of money supply follows AR(1) as we shown, Eq.(3.47) can be rewritten as:

$$p_t = m_t + \frac{\eta \rho_m}{1+\eta(1-\rho_m)} \Delta m_t \qquad (3.50)$$

Proof of Eq.(3.50)

- Eq.(3.47) can be rewriiten as:

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t\left(\Delta m_{t+k}\right)$$

$$= m_t + \frac{\eta}{1+\eta} \Delta m_{t+1} + \left(\frac{\eta}{1+\eta}\right)^2 \Delta m_{t+2} + \cdots$$

- Plugging AR(1) process for the growth rate of money supply yields:

$$-(m_t - p_t) = \frac{\eta \rho_m}{1+\eta} \Delta m_t + \left(\frac{\eta \rho_m}{1+\eta}\right)^2 \Delta m_t + \left(\frac{\eta \rho_m}{1+\eta}\right)^3 \Delta m_t + \cdots$$

$$= \left[\frac{\eta \rho_m}{1+\eta} + \left(\frac{\eta \rho_m}{1+\eta}\right)^2 + \left(\frac{\eta \rho_m}{1+\eta}\right)^3 + \cdots\right] \Delta m$$

- Dividing both sides of this by Δm_t yields:

$$-\frac{m_t - p_t}{\Delta m_t} = \frac{\eta \rho_m}{1+\eta} + \left(\frac{\eta \rho_m}{1+\eta}\right)^2 + \left(\frac{\eta \rho_m}{1+\eta}\right)^3 + \cdots \qquad (3.51)$$

- Multiplying both sides of Eq.(3.51) by $\eta \rho_m / (1+\eta)$ yields:

$$-\frac{m_t - p_t}{\Delta m_t} \frac{\eta \rho_m}{1+\eta} = \left(\frac{\eta \rho_m}{1+\eta}\right)^2 + \left(\frac{\eta \rho_m}{1+\eta}\right)^3 + \left(\frac{\eta \rho_m}{1+\eta}\right)^4 + \cdots \qquad (3.52)$$

- Subtracting Eq.(3.52) from Eq.(3.51) yields:

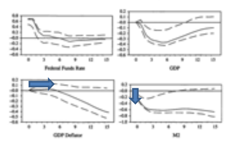
$$-\frac{m_t - p_t}{\Delta m_t} \left(1 - \frac{\eta \rho_m}{1+\eta}\right) = \frac{\eta \rho_m}{1+\eta}$$

- Dividing both sides of this by $1 - \eta \rho_m / (1+\eta)$ yields:

$$-(m_t - p_t) = \frac{\eta \rho_m}{1+\eta(1-\rho_m)} \Delta m_t$$

which equals to Eq.(3.50) obviously.

Fig. 10-1: Estimated Dynamic Response to a Monetary Policy Shock



Source: Christiano, Eichenbaum and Evans (1999)

- Eq.(3.50) shows that the price increases one by one to an exogenous increase in the money supply, as long as $\rho_m > 0$.
- Fig. 10-1 shows that the GDP deflator gradually falls with some lags when the money supply decreases.
- Thus, results shown by Eq.(3.50) is not consistent with data.

- Plugging the demand function for the real money balance

$$m_t - p_t = y_t - \eta \hat{i}_t$$

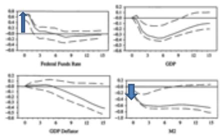
Into Eq. (3.50) yields:

$$\hat{i}_t = \frac{\rho_m}{1+\eta(1-\rho_m)} \Delta m_t \qquad (3.53)$$

where we have to pay attention to $y_t = 0$ is still applied.

- Eq.(3.53) shows that an increaese in the money supply induces an increase in the nominal interest rate, as long as $\rho_m > 0$.
- Is this consistent with data?

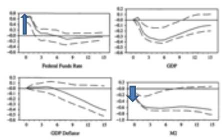
Fig: 10-1: Estimated Dynamic Response to a Monetary Policy Shock



Source: Christiano, Eichenbaum and Evans (1999)

- Data shows that there is a negative correration between the money supply and the nominal interest rate (Fig. 10-1).
- This negative correration is dubbed Liquidity Effect.
- That is, the nominal interest rate increases when the money decreaeses.

図10-1: Estimated Dynamic Response to a Monetary Policy Shock



Source: Christiano, Eichenbaum and Evans (1999)

- Because classical monetary model also cannot replicate the liquidity effect, it can be said that there is a limitation on analyzing actual monetary policy as long as we use classical monetary model.

Appendix Liquidity Effect

- Classical Monetary Model

$$\frac{M}{P} = L(Y, i)$$

1. An increase in the money supply not only increases current price (level) directly but also increases current price through an increase in future price based on rational expectation.
2. The real money balance decreases and the nominal interest rate increases.

Appendix Liquidity Effect

- Liquidity Effect

$$\frac{M}{P} = L(Y, i)$$

1. The money supply increases.
2. Because of sticky price, the real money balance increases. The nominal interest rate decreases.